

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES APPLICATION OF KAMAL TRANSFORM FOR SOLVING ABEL'S INTEGRAL EQUATION

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ABSTRACT

Abel's integral equation is a singular integral equation which appears in many branches of sciences such as physics, mechanics, radio astronomy, atomic scattering, X-ray radiography, electron emission and seismology. In this paper, we apply Kamal transform to solve Abel's integral equation and some numerical applications are given in application section to explain the effectiveness of Kamal transform for solving Abel's integral equation.

Keywords: *Abel's integral equation, Kamal transform, Inverse Kamal transform, Convolution theorem.*

I. INTRODUCTION

In 1823, Niels Henrik Abel studied the motion of particle on smooth curve lying on a vertical plane and described it in mathematical form in terms of Abel's integral equation as [1-2]

$$f(x) = \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \quad (1)$$

Here the kernel $K(x,t) = \frac{1}{\sqrt{x-t}}$ becomes ∞ at $t = x$, the function $f(x)$ is known function and the function $u(t)$ is unknown function.

In the modern time, integral transforms are of great value in the treatment of differential equations with constant or variable coefficients, partial differential equations with constant or variable coefficients, partial integro-differential equations, integral equations, integro-differential equations etc. Many scholars [3-26] used different integral transforms such as Laplace transform, Fourier transform, Kamal transform, Aboodh transform, Elzaki transform, Mohand transform, Hankel transform, Wavelet transform, Sumudu transform and solved many advanced problems of science, engineering and daily life.

In 2016, Abdelilah and Hassan [27] gave a new integral transform "Kamal transform" of the function $F(t)$ for $t \geq 0$ as

$$K\{F(t)\} = \int_0^\infty F(t) e^{-\frac{t}{v}} dt = G(v), \quad k_1 \leq v \leq k_2 \quad (2)$$

where operator K is called the Kamal transform operator.

Aggarwal et al. [28] gave a comparative study of Mohand and Kamal transforms. Aggarwal and Gupta [29] discussed the solution of linear Volterra integro-differential equations of second kind using Kamal transform. Abdelilah and Hassan [30] solved partial differential equations by applying Kamal transform. Aggarwal et al. [31] gave a new application of Kamal transform for solving linear Volterra integral equations. Gupta et al. [32] discussed the solution of linear partial integro-differential equations using Kamal transform. Application of Kamal transform for solving linear Volterra integral equations of first kind was given by Aggarwal et al. [33]. Aggarwal et al. [34]

used Kamal transform for solving population growth and decay problems. Aggarwal [35] defined Kamal transform of Bessel's functions.

In this paper, we solve Abel's integral equation using Kamal transform and explain all procedure by giving some numerical applications in application section.

II. SOME USEFUL PROPERTIES OF KAMAL TRANSFORM

2.1 Linearity property [28, 32-35]:

If Kamal transform of functions $F_1(t)$ and $F_2(t)$ are $G_1(v)$ and $G_2(v)$ respectively then Kamal transform of $[aF_1(t) + bF_2(t)]$ is given by $[aG_1(v) + bG_2(v)]$, where a, b are arbitrary constants.

2.2 Change of scale property [28, 35]:

If Kamal transform of function $F(t)$ is $G(v)$ then Kamal transform of function $F(at)$ is given by $\frac{1}{a}G(av)$.

2.3 Shifting property [28]:

If Kamal transform of function $F(t)$ is $G(v)$ then Kamal transform of function $e^{at}F(t)$ is given by $G\left(\frac{v}{1-av}\right)$.

2.4 Kamal transform of the derivatives of the function $F(t)$ [27-29, 31-32, 34-35]:

If $K\{F(t)\} = G(v)$ then

- $K\{F'(t)\} = \frac{1}{v}G(v) - F(0)$
- $K\{F''(t)\} = \frac{1}{v^2}G(v) - \frac{1}{v}F(0) - F'(0)$
- $K\{F^{(n)}(t)\} = \frac{1}{v^n}G(v) - \frac{1}{v^{n-1}}F(0) - \frac{1}{v^{n-2}}F'(0) \dots - F^{(n-1)}(0)$

2.5 Convolution theorem for Kamal transforms [28-29, 31-33]:

If Kamal transform of functions $F_1(t)$ and $F_2(t)$ are $G_1(v)$ and $G_2(v)$ respectively then Kamal transform of their convolution $F_1(t) * F_2(t)$ is given by

$$K\{F_1(t) * F_2(t)\} = K\{F_1(t)\}K\{F_2(t)\}$$

$\Rightarrow K\{F_1(t) * F_2(t)\} = G_1(v)G_2(v)$, where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)F_2(x)dx = \int_0^t F_1(x)F_2(t-x)dx$$

III. KAMAL TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [27-29, 31-35]

Table: 1

S.N.	$F(t)$	$K\{F(t)\} = G(v)$
1.	1	v
2.	t	v^2
3.	t^2	$2!v^3$
4.	$t^n, n \in N$	$n!v^{n+1}$
5.	$t^n, n > -1$	$\Gamma(n + 1)v^{n+1}$
6.	e^{at}	$\frac{v}{1 - av}$
7.	$\sin at$	$\frac{av^2}{1 + a^2v^2}$
8.	$\cos at$	$\frac{v}{1 + a^2v^2}$
9.	$\sinh at$	$\frac{av^2}{1 - a^2v^2}$
10.	$\cosh at$	$\frac{v}{1 - a^2v^2}$
11	$J_0(t)$	$\frac{v}{\sqrt{(1 + v^2)}}$
12	$J_1(t)$	$1 - \frac{1}{\sqrt{(1 + v^2)}}$

IV. INVERSE KAMAL TRANSFORM [28-29, 31-35]

If $K\{F(t)\} = G(v)$ then $F(t)$ is called the inverse Kamal transform of $G(v)$ and mathematically it is defined as $F(t) = K^{-1}\{G(v)\}$, where K^{-1} is the inverse Kamal transform operator.

V. LINEARITY PROPERTY OF INVERSE KAMAL TRANSFORMS [34]

If $K^{-1}\{H(v)\} = F(t)$ and $K^{-1}\{I(v)\} = G(t)$ then
 $K^{-1}\{aH(v) + bI(v)\} = aK^{-1}\{H(v)\} + bK^{-1}\{I(v)\}$
 $\Rightarrow K^{-1}\{aH(v) + bI(v)\} = aF(t) + bG(t)$, where a, b are arbitrary constants.

VI. INVERSE KAMAL TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS
[28-29, 31-35]

Table: 2

S.N.	$G(v)$	$F(t) = K^{-1}\{G(v)\}$
1.	v	1
2.	v^2	t
3.	v^3	t^2
		$\frac{2!}{2!}$
4.	$v^{n+1}, n \in N$	$\frac{t^n}{n!}$
5.	$v^{n+1}, n > -1$	$\frac{n!}{\Gamma(n+1)}$
6.	$\frac{v}{1-av}$	e^{at}
7.	$\frac{v^2}{1+a^2v^2}$	$\frac{\sin at}{a}$
8.	$\frac{v}{1+a^2v^2}$	$\cos at$
9.	$\frac{v^2}{1-a^2v^2}$	$\frac{\sinh at}{a}$
10.	$\frac{v}{1-a^2v^2}$	$\cosh at$
11.	$\frac{1}{\sqrt{1+v^2}}$	$J_0(t)$
12.	$1 - \frac{1}{\sqrt{1+v^2}}$	$J_1(t)$

VII. KAMAL TRANSFORM FOR SOLVING ABEL’S INTEGRAL EQUATION:

In this section, we present Kamal transform for solving Abel’s integral equation.

Taking Kamal transform of both sides of (1), we have

$$K\{f(x)\} = K\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t) dt\right\}$$

$$\Rightarrow K\{f(x)\} = K\{x^{-1/2} * u(x)\} \tag{3}$$

Applying convolution theorem of Kamal transform in (3), we have

$$K\{f(x)\} = K\{x^{-1/2}\}K\{u(x)\}$$

$$\Rightarrow K\{f(x)\} = \sqrt{\pi}v^{1/2}K\{u(x)\}$$

$$\Rightarrow K\{u(x)\} = \frac{1}{\sqrt{\pi}v^{1/2}}K\{f(x)\}$$

$$\Rightarrow K\{u(x)\} = \frac{1}{\pi v}[\sqrt{\pi}v^{1/2}K\{f(x)\}]$$

$$\Rightarrow K\{u(x)\} = \frac{1}{\pi v}[K\{x^{-1/2}\}K\{f(x)\}]$$

$$\Rightarrow K\{u(x)\} = \frac{1}{\pi v}K\{x^{-1/2} * f(x)\}$$

$$\Rightarrow K\{u(x)\} = \frac{1}{\pi v}\left[K\left\{\int_0^x \frac{1}{\sqrt{x-t}} f(t) dt\right\}\right]$$

$$\Rightarrow K\{u(x)\} = \frac{1}{\pi v}K\{F(x)\} \tag{4}$$

where $F(x) = \int_0^x \frac{1}{\sqrt{x-t}} f(t) dt$ (5)

Now applying the property, Kamal transform of derivative of the function, on (5), we have

$$K\{F'(x)\} = \frac{1}{v}K\{F(x)\} - F(0)$$

$$\Rightarrow K\{F'(x)\} = \frac{1}{v}K\{F(x)\}$$

$$\Rightarrow K\{F(x)\} = vK\{F'(x)\} \tag{6}$$

Now from (4) and (6), we have

$$K\{u(x)\} = \frac{1}{\pi}K\{F'(x)\} \tag{7}$$

Applying inverse Kamal transform on both sides of (7), we get

$$u(x) = \frac{1}{\pi}F'(x) = \frac{1}{\pi} \frac{d}{dx} F(x) \tag{8}$$

Using (5) in (8), we have

$$u(x) = \frac{1}{\pi} \left[\frac{d}{dx} \int_0^x \frac{1}{\sqrt{x-t}} f(t) dt \right] \tag{9}$$

which is the required solution of (1).

VIII. APPLICATIONS

In this section, we present some numerical applications to explain the complete procedure of solving Abel’s integral equation using Kamal transform.

8.1 Consider the Abel’s integral equation:

$$x = \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \tag{10}$$

Taking Kamal transform of both sides of (10), we have

$$K\{x\} = K\left\{ \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \right\}$$

$$\Rightarrow v^2 = K\{x^{-1/2} * u(x)\} \tag{11}$$

Applying convolution theorem of Kamal transform in (11), we have

$$v^2 = K\{x^{-1/2}\}K\{u(x)\}$$

$$\Rightarrow v^2 = \sqrt{\pi}v^{1/2}K\{u(x)\}$$

$$\Rightarrow K\{u(x)\} = \frac{v^{3/2}}{\sqrt{\pi}} \tag{12}$$

Applying inverse Kamal transform on both sides of (12), we get

$$u(x) = \frac{1}{\sqrt{\pi}}K^{-1}\{v^{3/2}\}$$

$$\Rightarrow u(x) = \frac{2}{\pi}x^{1/2} \tag{13}$$

which is the required solution of (10).

8.2 Consider the Abel’s integral equation:

$$1 + x + x^2 = \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \tag{14}$$

Taking Kamal transform of both sides of (14), we have

$$K\{1\} + K\{x\} + K\{x^2\} = K\left\{ \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \right\}$$

$$\Rightarrow v + v^2 + 2v^3 = K\{x^{-1/2} * u(x)\} \tag{15}$$

Applying convolution theorem of Kamal transform in (15), we have

$$v + v^2 + 2v^3 = K\{x^{-1/2}\}K\{u(x)\}$$

$$\Rightarrow v + v^2 + 2v^3 = \sqrt{\pi}v^{1/2}K\{u(x)\}$$

$$\Rightarrow K\{u(x)\} = \frac{1}{\sqrt{\pi}} [v^{1/2} + v^{3/2} + 2v^{5/2}] \quad (16)$$

Applying inverse Kamal transform on both sides of (16), we get

$$\begin{aligned} u(x) &= \frac{1}{\sqrt{\pi}} K^{-1}\{v^{1/2} + v^{3/2} + 2v^{5/2}\} \\ \Rightarrow u(x) &= \frac{1}{\sqrt{\pi}} [K^{-1}\{v^{1/2}\} + K^{-1}\{v^{3/2}\} + 2K^{-1}\{v^{5/2}\}] \\ \Rightarrow u(x) &= \frac{1}{\pi} [x^{-1/2} + 2x^{1/2} + \frac{8}{3}x^{3/2}] \end{aligned} \quad (17)$$

which is the required solution of (14).

8.3 Consider the Abel's integral equation:

$$3x^2 = \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \quad (18)$$

Taking Kamal transform of both sides of (18), we have

$$\begin{aligned} 3K\{x^2\} &= K\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t) dt\right\} \\ \Rightarrow 6v^3 &= K\{x^{-1/2} * u(x)\} \end{aligned} \quad (19)$$

Applying convolution theorem of Kamal transform in (19), we have

$$\begin{aligned} 6v^3 &= K\{x^{-1/2}\}K\{u(x)\} \\ \Rightarrow 6v^3 &= \sqrt{\pi}v^{1/2}K\{u(x)\} \\ \Rightarrow K\{u(x)\} &= \frac{6}{\sqrt{\pi}}v^{5/2} \end{aligned} \quad (20)$$

Applying inverse Kamal transform on both sides of (20), we get

$$\begin{aligned} u(x) &= \frac{6}{\sqrt{\pi}} K^{-1}\{v^{5/2}\} \\ \Rightarrow u(x) &= \frac{8}{\pi} x^{3/2} \end{aligned} \quad (21)$$

which is the required solution of (18).

8.4 Consider the Abel's integral equation:

$$\frac{4}{3}x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \quad (22)$$

Taking Kamal transform of both sides of (22), we have

$$\begin{aligned} \frac{4}{3}K\{x^{3/2}\} &= K\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t) dt\right\} \\ \Rightarrow \sqrt{\pi}v^{5/2} &= K\{x^{-1/2} * u(x)\} \end{aligned} \quad (23)$$

Applying convolution theorem of Kamal transform in (23), we have

$$\begin{aligned} \sqrt{\pi}v^{5/2} &= K\{x^{-1/2}\}K\{u(x)\} \\ \Rightarrow \sqrt{\pi}v^{5/2} &= \sqrt{\pi}v^{1/2}K\{u(x)\} \\ \Rightarrow K\{u(x)\} &= v^2 \end{aligned} \quad (24)$$

Applying inverse Kamal transform on both sides of (24), we get

$$\begin{aligned} u(x) &= K^{-1}\{v^2\} \\ \Rightarrow u(x) &= x \end{aligned} \quad (25)$$

which is the required solution of (22).

8.5 Consider the Abel's integral equation:

$$2\sqrt{x} + \frac{8}{3}x^{3/2} = \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \quad (26)$$

Taking Kamal transform of both sides of (26), we have

$$\begin{aligned} 2K\{x^{1/2}\} + \frac{8}{3}K\{x^{3/2}\} &= K\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t) dt\right\} \\ \Rightarrow \sqrt{\pi}v^{3/2} + 2\sqrt{\pi}v^{5/2} &= K\{x^{-1/2} * u(x)\} \end{aligned} \quad (27)$$

Applying convolution theorem of Kamal transform in (27), we have

$$\begin{aligned}\sqrt{\pi}v^{3/2} + 2\sqrt{\pi}v^{5/2} &= K\{x^{-1/2}\}K\{u(x)\} \\ \Rightarrow \sqrt{\pi}v^{3/2} + 2\sqrt{\pi}v^{5/2} &= \sqrt{\pi}v^{1/2}K\{u(x)\} \\ \Rightarrow K\{u(x)\} &= v + 2v^2\end{aligned}\quad (28)$$

Applying inverse Kamal transform on both sides of (28), we get

$$\begin{aligned}u(x) &= K^{-1}\{v\} + 2K^{-1}\{v^2\} \\ \Rightarrow u(x) &= 1 + 2x\end{aligned}\quad (29)$$

which is the required solution of (26).

8.6 Consider the Abel's integral equation:

$$\frac{3}{8}\pi x^2 = \int_0^x \frac{1}{\sqrt{x-t}} u(t) dt \quad (30)$$

Taking Kamal transform of both sides of (30), we have

$$\begin{aligned}\frac{3}{8}\pi K\{x^2\} &= K\left\{\int_0^x \frac{1}{\sqrt{x-t}} u(t) dt\right\} \\ \Rightarrow \frac{3}{4}\pi v^3 &= K\{x^{-1/2} * u(x)\}\end{aligned}\quad (31)$$

Applying convolution theorem of Kamal transform in (31), we have

$$\begin{aligned}\frac{3}{4}\pi v^3 &= K\{x^{-1/2}\}K\{u(x)\} \\ \Rightarrow \frac{3}{4}\pi v^3 &= \sqrt{\pi}v^{1/2}K\{u(x)\} \\ \Rightarrow K\{u(x)\} &= \frac{3}{4}\sqrt{\pi}v^{5/2}\end{aligned}\quad (32)$$

Applying inverse Kamal transform on both sides of (32), we get

$$\begin{aligned}u(x) &= \frac{3}{4}\sqrt{\pi}K^{-1}\{v^{5/2}\} \\ \Rightarrow u(x) &= x^{3/2}\end{aligned}\quad (33)$$

which is the required solution of (30).

IX. CONCLUSION

In this paper, we have successfully discussed the application of Kamal transform for solving Abel's integral equation. The given numerical applications in the application section explain the whole procedure of this scheme. The results show that Kamal transform is a powerful integral transform for handling Abel's integral equation. In future, Kamal transform can be used for solving other singular integral equations and their systems.

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